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DISCUSSION OF HIGH-VELOCITY FLOW IN OPEN CHANNELS: A SYMPOSIUM

(Published in November, 1949)

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HYDRAULICS DIVISION

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DISCUSSION

PAUL BAUMANN,[32] M. ASCE.—In the years after 1929 dams have risen from maximum heights of from 300 ft to 700 ft and still the trend toward greater heights persists.

Since spillways are indispensable appurtenances of dams, it is obvious that the maximum velocity of water discharging from them has increased in direct relation to the height of the structure above the stream bed. Simultaneously flood channels below the spillway of debris barriers, located on the cones below canyons of the coastal mountain range of Southern California, have been designed and constructed for water velocities far above critical—that is, for velocities of the order of 70 ft per sec. The slopes of these channels often exceed 10%.

The Symposium on supercritical flow in open channels is most timely, therefore, particularly in view of the vagueness that existed until recent years in connection with the mechanics of this phenomenon. However, this Symposium is to be welcomed by the profession, not only because it is timely, but also (and indeed, particularly) because it is presented masterfully, vividly, and fascinatingly, in a manner that cannot but help stimulate the interest of the hydraulic experimenter and designer. The questions so freely admitted as being still unanswered by the authors will thereby be moved into the focus of broader attention, and the more complete, as well as exact, solution of the problem may well be much advanced.

The writer, being in charge of the general direction of hydraulic research of the Los Angeles County (Calif.) Flood Control District, naturally kept in close touch with the California Institute of Technology at Pasadena throughout the model tests and their interpretation so clearly presented in the papers by Messrs. Knapp and Ippen. The testing apparatus and plant were financed by the District and in part constructed by its forces. In the design of the model and the testing work itself the institute was assisted by district personnel. Among them Wilbert W. Baustian of the Hydraulic Division contributed many constructive ideas.

The validity of the approach to the problem of supercritical flow of water on the principle of supersonic flow of gases has no doubt been established as a result of the tests and their mathematical analysis. The abrupt and sharp boundary lines (formed by the waves due to a disturbance, such as a sudden change from a straight to a curved alinement of a channel) cannot help but suggest a striking similarity to the well-known V-shaped "wake" which forms at the point of a projectile, moving through air at a speed greater than sound.

NOTE.—This Symposium was published in November, 1949, *Proceedings*. The numbering of footnotes, equations, tables, and illustrations in this Separate is a continuation of the consecutive numbering used in the original Symposium.

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The oscillatory movement of the water is likewise apparent in all cases in which the radial water prism loses its balance. This happens when water enters a circular curve without suitable transition.

Certain approximations had to be made to facilitate solution of the problem in the form of design equations. Of those, the one referred to by Mr.

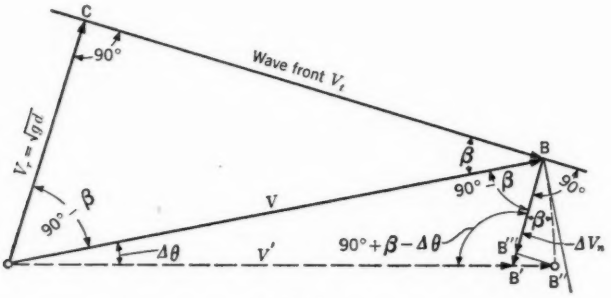

FIG. 64

Knapp in his discussion of Eq. 2 is notable. However, it is not Eq. 2 itself that is an approximation, but rather that which follows as a result of the substitution, $V_n = V \sin \beta$, which leads to Eq. 30. It corresponds to Eq. 6. The apparent discrepancy could mathematically be justified for only very small wave angles β which is not necessarily the case. The approximation in question can be illustrated by Fig. 64. Whereas Eq. 6 is based on V changing to V', which is represented by AB', Fig. 64, and therefore $-\Delta V_n$ is represented by BB', Eq. 30 is based on V being constant. Therefore $-\Delta V_n$ is represented by BB''', and the tangential component represented by B''B''' is neglected. Since $\Delta\theta$ is small as compared to β, this neglect, although contrary to the basic premise, is well within the tolerance of error for design purposes.

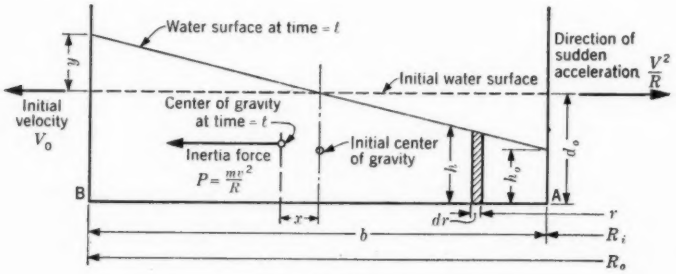

FIG. 65

For the sake of comparison with the results set forth in the papers, equations for the maximum superelevation and the distance from the point of maximum superelevation to the beginning of the curve are derived as follows: To represent the phenomena simply, the assumption is made that water in a tank of a rectangular section, such as a channel, is being moved at the constant velocity V_o

(see Fig. 65). At a time $t = 0$ the direction of movement of the tank is suddenly reversed and the tank is accelerated in the opposite direction at a constant acceleration of V^2/R (symbol R denotes curve radius). The water will then (because of the inertia forces) flow in the opposite direction of motion and will bank up against the respective wall of the tank to a maximum superelevation, and then flow back. This motion is comparable with a harmonic oscillation under the influence of a constant force, P, acting either in the direction of flow or opposite to it. The differential equation describing this motion in the direction of P is

$$\frac{-m\,d^2x}{dt^2} - K\,x + P = 0 \qquad (50)$$

or

$$\frac{+d^2x}{dt^2} + \frac{K}{m}(x - a) = 0 \qquad (51)$$

in which $a = P/K$.

The solution of Eq. 51 is known, and is

$$x - a = A \sin \alpha\, t + B \cos \alpha\, t \qquad (52)$$

Determining the constants of integration A and B from the boundary conditions,

$$B = -a \qquad (53a)$$

and

$$A = \frac{V_o}{\alpha} \qquad (53b)$$

Hence, the equation of motion reads

$$x - a = \frac{V_o}{\alpha} \sin \alpha\, t - a \cos \alpha\, t \qquad (54)$$

The velocity at any instant, then, is

$$\frac{dx}{dt} = \alpha \left(\frac{V_o}{\alpha} \cos \alpha\, t + a \sin \alpha\, t \right) \qquad (55)$$

and the acceleration at any instant is

$$\frac{d^2x}{dt^2} = -\alpha^2 \left(\frac{V_o}{\alpha} \sin \alpha\, t - a \cos \alpha\, t \right) \qquad (56)$$

In Eq. 56 the term in the parenthesis may be seen to equal $x - a$, and the constant α, therefore, is equal to $\sqrt{K/m}$.

From conditions of equilibrium of the water in the tank (Fig. 65) it follows that the differential pressure P_x between inside and outside walls at any instant is equal to $2\,w\,y\,d_o$. Since x represents the horizontal displacement of the center of gravity of the oscillating prism, the superelevation y can be expressed as a function of x—namely,

$$y = 6\,d_o \frac{x}{b} \qquad (57)$$

Introducing

$$P_x = \frac{12\,w\,d_o^2}{b}\,x \qquad (58a)$$

the constant K then equals $\frac{12\,w\,d_o^2}{b}$, and, since $a = P/K$, $P = m\,v^2/R$, and $m = \frac{w\,b\,d_o}{g}$:

$$a = \frac{v^2\,b^2}{12\,R\,g\,d_o} \qquad (58b)$$

Introducing these values in Eq. 54,

$$x = \frac{v_o\,b}{\sqrt{12}\,\sqrt{g\,d_o}} \sin \alpha\,t + \frac{v^2\,b^2}{12\,R\,g\,d_o}\,(1 - \cos \alpha\,t) \qquad (59)$$

The first member on the right side of Eq. 59 represents the influence of initial velocity whereas the second member represents the influence of constant acceleration during oscillation. Introducing x into Eq. 57,

$$y = d_o \left[\frac{\sqrt{3}\,V_o}{\sqrt{g\,d_o}} \sin \alpha\,t + \frac{V^2\,b}{2\,R\,g\,d_o}\,(1 - \cos \alpha\,t)\right] \qquad (60)$$

In studying this result it must be kept in mind that Eq. 60 is based on the assumption that the entire mass is concentrated in the center of gravity. In other words, the water prism is treated as a mass point. This is not the case in reality, and instantaneous acceleration at the beginning of the curve is only partly taking place with water in a channel. The approximation is on the safe side, however. In the case of water confined in a tank, instantaneous acceleration is very nearly possible.

An equation for y similar to Eq. 60 could be derived for the motion in the direction opposite to force P. This would lead to smaller values of y and therefore would not be a governing consideration for design purposes. It is obvious that v_o may become important.

Checking back on the boundary conditions, it follows that the premises upon which the analysis is based are satisfied. It is also evident that $\frac{v}{\sqrt{g\,d_o}}$ (which is the cosecant of β, the wave angle) insists on appearing as a parameter. When $v_o = 0$ the maximum superelevation occurs for $\alpha\,t = \pi$. Hence,

$$x = \frac{v^2\,b^2}{6\,R\,g\,d_o} \qquad (61a)$$

and

$$y_{max} = \frac{v^2\,b}{R\,g} \qquad (61b)$$

The period of oscillation T follows from

$$T = \frac{2\,\pi}{\alpha} = 2\,\pi\,\sqrt{\frac{m}{K}} = \frac{\pi\,b}{\sqrt{3}\,\sqrt{g\,d_o}} \qquad (62)$$

The time required for the water to reach its first maximum superelevation from the neutral position would then be one half the values found by Eq. 62, and the distance from the beginning of the curve to that point would be:

$$D = \frac{v \pi b}{\sqrt{12}\sqrt{g d_o}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (63)$$

From this analysis it may be readily recognized why no oscillations develop in correctly banked channels. The driving force P is exactly balanced at all instants by the component of a gravitational force, due to a sloping channel bottom. Hence, in Eq. 59 the first member disappears because of no initial radial velocity, and the second member disappears because of a third member which is equal to it and opposite in sign. The displacement x (Fig. 65) of the center of gravity then remains zero throughout the movement of water around the curve.

It is interesting to note that Eq. 8 leads to Eq. 61*b* for long radius curves—that is, for the case in which the arc of the outside wall may be approximated by the tangent and θ is substituted for $d\theta$.

Eq. 31 is an approximation of Eq. 8. For long radius curves, and when θ is small in relation to β, Eq. 31 may also be written in finite terms (see Fig. 64):

$$h = \frac{v^2}{g} \sin \beta \cos \beta \, \theta = \frac{(V^2 - g d_o) b}{R g} = \frac{V_t^2 b}{R g} \quad \ldots\ldots\ldots\ldots (64)$$

Thus, instead of V its tangential component V_t appears in Eq. 64. Obviously, the discrepancy between the two formulas (Eqs. 31 and 8) grows with the wave angle β.

An interesting comparison of Eqs. 32*b* and 61*b* has been made by C. A. Hart, Assoc. M. ASCE, by plotting maximum superelevations observed during 80 model runs at the California Institute of Technology and 10 runs on the Puddingstone Flood Channel prototype. Values by Eq. 32*b* (minus the measured value) were plotted as abscissas against values by Eq. 61*b* (minus the measured values) as ordinates. By analyzing the scatter in quadrants, formed by 45° axes, of the chart it was observed that 45 of the 80 model observations came closer to checking Eq. 32*b* and that 6 of the 10 observations on the prototype came closer to checking Eq. 61*b*.

The comparison between Eq. 33 and Eq. 63 may best be shown by a numerical example. Let: $Q = 5{,}000$ cu ft per sec; $b = 20$ ft; $d_o = 5$ ft; $A = 100$ ft^2; $R = 500$ ft; $v = Q/A = 50$ ft per sec; $\tan \beta_o = \dfrac{\sin \beta_o}{\cos \beta_o} = \dfrac{\sqrt{g d_o}}{\sqrt{v^2 - g d_o}} = 0.263$; and from Eq. 33, $\theta_o = \tan^{-1} \dfrac{20}{510 \times 0.263} = \tan^{-1} 0.15$. Hence $D = R\theta = 74.30$ ft; and from Eq. 63, $D = \dfrac{50 \times 3.14 \times 20}{\sqrt{12} \times 12.68} = 71.50$ ft. The difference of $74.30 - 71.50 = 2.80$ ft corresponds to a difference of 0.056 sec in time.

All the model tests at the California Institute of Technology were conducted on channels of uniform bottom width and uniform slopes for trapezoidal channels. For banked rectangular channels the bottom was straight in a

radial direction. This is a satisfactory approximation for a width b which is small relative to the center-line radius of the channel. Actually, the water surface for balanced flow is always a curve, the equation of which reads (see Fig. 65):

$$h = \frac{v^2}{g} \log_e \frac{r}{R_i} + h_o \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (65)$$

Eq. 65 is the equation of a free vortex.

In a banked channel the bottom must be parallel to the water surface for balanced flow (that is, the depth must remain constant); therefore, Eq. 65 also holds true for the curvature of the bottom if h_o is replaced by a value that varies between 0 and $-\frac{v^2}{g} \log_e \frac{R_o}{R_i}$ and which causes the curve to pass through any point between A and B in Fig. 65. The wisdom as to which point should be chosen is governed by conditions in the field. Whenever practicable it is advantageous to use such a point between A and B for which the change in position of the center of gravity of the prism is a minimum.

In the fall of 1948, the Los Angeles Flood Control District completed a section of a major rectangular flood channel 50 ft wide. Spiral transitions were used in the design. As a result of this experience Mr. Knapp's apprehension regarding such transitions has not been found to be justified in regard to difficulties in design, layout, or construction. Hydraulically, a spiral transition at least one half of a design wave length in length is superior to compound curves, particularly for rates of flow considerably smaller than the design flow.

The writer has been a strong believer in, and proponent for, banked rectangular channels with spiral transions for supercritical flow. Large-scale model tests conducted in the summer of 1947 by the United States Corps of Engineers, Los Angeles District, have shown remarkable confirmation of the small-scale tests at the California Institute of Technology and have strengthened the writer's confidence in the value of the latter.

The writer has often heard engineers express doubt as to the performance of banked channels for flows smaller than design flow. Fear was expressed that the flow would concentrate on the inside of the curve, forcing it to go over the top of the wall. A study of this problem will readily show that, because of the influence of the inertia force, $P = \frac{m\,v^2}{r}$, this cannot happen. This theory has been fully confirmed by model tests.

As to sills as described in Mr. Knapp's paper, there can be no doubt that, scientifically, they are an elegant suggestion. Practically, however, they have limited usefulness in flood channels mainly because of the almost unavoidable debris problem involved.

The disadvantages of trapezoidal channels can be greatly alleviated through the provision of vertical parapet walls at the top of the slopes. Of course, banking is not practicable.

In the foregoing the term "transition" was used to describe the change from a straight to a curved section of a channel without a change in width. In the papers on the design of channel expansions and contractions for supercritical flow the term "transition" refers to a sudden or gradual change in width.

These papers are closely related to the ones previously discussed, particularly to the one by Mr. Ippen. The simplicity of the mathematical solution by Messrs. Rouse, Bhoota, and Hsu of a problem of such complexity is remarkable. The paper by Messrs. Ippen and Dawson brings out the fact that for supercritical flow contractions must be designed with particular care. Fig. 40 is most helpful in demonstrating the significance of high Froude numbers relative to the most advantageous length and configuration of the transition. It shows clearly that gradual transitions commonly used for subcritical flow or low Froude numbers are not necessarily desirable. The formulas and graphs presented in these papers will be of considerable help to the designers. The Los Angeles Flood Control District has carefully avoided sudden changes in width of flood channels and, wherever possible, has confined gradual transitions to straight sections of channels. The length of these transitions has been determined by the wave angle theory. No trouble has ever developed under these conditions.

Concluding, the writer believes that the design of flood channels is in quite a different category from the design and installation of hydraulic machinery in connection with clear, well controlled water. Changes often occur in flood channels, such as in the discharge of storm drains into them, which may cause more serious disturbance than plain circular curves. Liberal freeboard is therefore advisable. In this case it is truly a matter of providing for a factor of ignorance.

N. N. BHANDARI.[33]—The phenomenon discussed in the Symposium is particularly interesting in the light of the viewpoints from which the different authors have studied it. This problem is of great importance to the irrigation engineers in India as an increasing number of works is being constructed which involve fluming or constriction, and then expansion, of channels.

Experiments have been conducted by J. Nikuradse[34] on pressure flow in smooth channels with various angles between the walls. It was found that the symmetrical flow was stable when the channel was convergent or slightly divergent—even with a divergence of 4° there was no trace of instability. The velocity distributions for divergences of 5°, 6°, and 8° were not symmetrical about the middle plane. No region of back flow at either wall was evident with a divergence of 5°; separation of the forward flow in the boundary layer along one of the walls was just beginning to occur at 5°. Mr. Nikuradse found that the first occurrence happened between 4.8° and 5.1°, but could not be specified more exactly. At 6° the separation became apparent from the measurements taken; at which wall the separation occurred was a question settled by incidental changes in pressure, and once it occurred the pressure gradients were altered. For divergences of 5° and 6° there was no apparent tendency for separation to occur at the second wall, and the symmetrical flow once established seemed to be quite stable. A tendency appeared for the symmetrical distribution to switch over from wall to wall at 8°; each con-

[33] Asst. Secy., Central Board of Irrig., Simla, India.

[34] "Modern Developments in Fluid Dynamics," by S. Goldstein, Clarendon Press, Oxford, 1938, Vol. II, pp. 371–373.

figuration was usually maintained for some time—long enough to allow measurements to be taken before a change occurred.

To the writer the foregoing observations are of utmost importance. This kind of action of the flow downstream from the expansions has generally been observed in both experimental flumes and prototype channels.

The result of this action has also been observed in the asymmetrical shape of the scour hole created downstream from such structures. Attempts have been made to control the flow in such cases by "bowing" the downstream cross section of these structures by making the downstream beds curved across the width instead of leaving it flat. This idea of "bowing" the cross section was first introduced by A. M. R. Montagu[35] in 1934. The bowed bed concentrates the flow longitudinally in the center, makes the scour hole symmetrical, and reduces the side scours; but it increases the depth of scour hole in the center considerably, accentuating the dangers to the stability of the structure. To guard against this action additional precautionary measures in the form of staggered blocks,[36] baffles, etc., must be incorporated in the structure, further complicating an already complicated problem. No one seems to have proceeded on the lines indicated by Mr. Nikuradse to design the expansion. The expansion required may not actually be 5°. It would naturally depend, to a large extent, on the discharge required to be passed through the flumed structure. Unfortunately there is no hydraulic laboratory at the disposal of the writer in which he can complete an ideal simple design of a flumed structure on the foregoing principles. The flinging jets interfere with each other and create a number of wavelets, as indicated in the Symposium in some of the illustrations and photographs of flow in expansions. The writer would like to suggest further experimental work on the lines indicated herein, to be conducted by those who have the facilities to do so. This would be a great contribution to the knowledge of the profession.

T. BLENCH,[37] M. ASCE.—The Symposium is welcome as a logical extension of the methods developed by Julian Hinds,[38] M. ASCE. It relies on the Bernoulli equation and the momentum (hydraulic jump) equations under simplifying assumptions concerning the nature of the flow, and it uses graphical methods of solution; but a special assumption of no energy loss over a sudden rise of surface elevation—unnecessary in Mr. Hinds' work—seems to have potentialities for producing considerable error in certain cases. The value of graphical solutions based on simplifying assumptions has been well demonstrated in India, where Mr. Hinds' methods have been used since 1934 in the form of officially published graphs. The result has been a marked advance in the design of flumes and drops in erodible channels; the writer attributes this advance largely to the fact that every engineer was able to create a mental picture of ideal behavior as a basis for comparison with actual occurrences.

[35] "Fluming," by A. M. R. Montagu, *Publication No. 6*, Central Board of Irrig., Simla, India, 1934.

[36] "Protection Against Scour of River and Canal Works," by N. N. Bhandari and H. L. Uppal, *Paper No. 214*, Punjab Eng. Congress, 1938.

[37] Cons. Engr. and Associate Prof., Civ. Eng., Univ. of Alberta, Edmonton, Alta., Canada.

[38] "The Hydraulic Jump and Critical Depth in the Design of Hydraulic Structures," by Julian Hinds, *Engineering News-Record*, Vol. LXXXV, July–December, 1920, p. 1034.

The authors' line of attack seems to have "paid dividends" already by indicating the first approximation to the production of interference patterns that will have a small maximum wave amplitude. This should put an end to radically incorrect designs and should reduce model experimentation to the improvement of basically correct models. Because of the basic value of the work, the writer is not perturbed by the rather poor numerical agreement that can occur between theory and experiment as in Fig. 38.

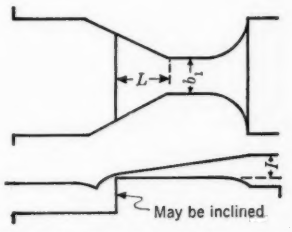

FIG. 66.—DIAGRAM OF SPREADING FLOOR DROP

The writer's principal practical interest is in Fig. 57 as a possible confirmation of the length, L, required for the spreading floor of a fairly new type of nonscouring fall.[39,40] Fig. 57 does not seem to agree well with model results obtained in India, and the question arises as to the sensitivity of the surface conformation of a hypercritical expansion to such factors as initial velocity distribution and friction, which the simple theory ignores. Fig. 66 illustrates the spreading floor design, in principle. Instead of starting a chute and the divergence simultaneously from the flumed throat, the crest level is continued a length, L, into the divergence until almost regime breadth of the downstream channel is reached, then the drop is provided. The spreading floor may slope upward in the downstream direction to insure a definite plunging of the water surface over the drop when the difference of level between the upstream

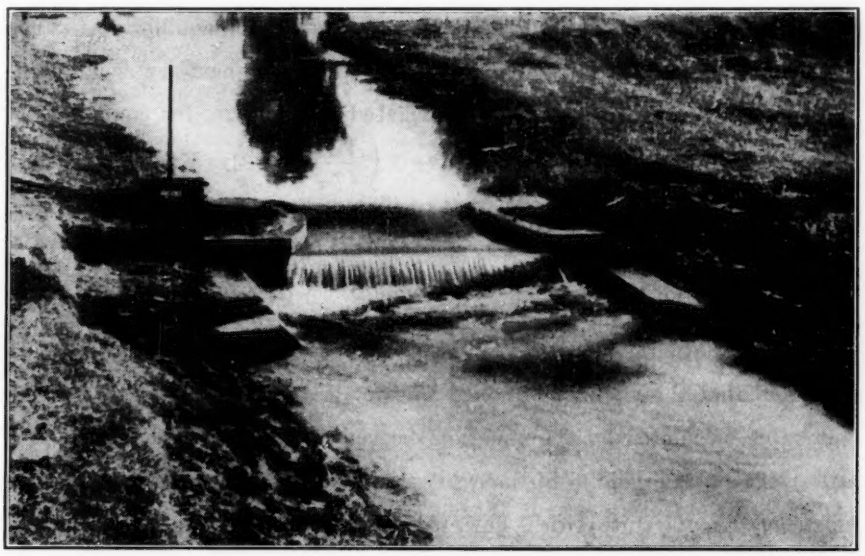

FIG. 67.—SPREADING FLOOR MODEL OF ABOUT 20 CU FT PER SEC

[39] "Scour in Alluvium Below Falls," by T. Blench, Mushtaq Ahmad, and Nazir Ahmad, Rept. on the 22d Meeting of the International Assn. for Hydraulic Structures Research, Stockholm, 1948.

[40] "Design of Canal Falls," by T. Blench and Mushtaq Ahmad, *Research Publication*, Irrig. Research Inst., West Punjab, Pakistan, Vol. 11, No. 26.

and the downstream flow is small. Side-wall divergences of 1:2 are satisfactory. (The principal object of the design is to take advantage of the rapid divergence that can occur in hypercritical flow, because there is acceleration; the failure of subcritical divergences—from the viewpoint of scour—arises from the fact that they require deceleration and the friction that could produce it is too small to permit a divergence rate usually considered economical.) The length L was determined from a range of small model experiments to produce a practically uniform depth of flow just before the drop—not more than 5% variation. These experiments showed that L/b_1 was very dependent on H/b_1—in fact, 0.9, 1.0, and 1.4 correspond to 0.075, 2.4, and 2.9. In Fig. 57, taking F_1 as 1.0—which seems to correspond reasonably with the conditions of Fig. 66—and using $x/b_1 = 0.9$, the mean rate of side divergence up to that value of x/b_1 is 1:2.25 with a central depth of about 50% greater than the side depth; but the Indian experiments show that with the sharper divergence of 1:2 there would be practically uniform depth all the way across. Fig. 67 shows a model of about 20 cu ft per sec working; the spreading floor had a very slight upward slope.

The writer would appreciate any information on a counterpart of Fig. 57 for flow emerging from a parallel flume throat, with divergence walls arranged to insure the rapid assumption of uniform depth at right angles to the center line.

CLARENCE A. HART,[41] ASSOC. M. ASCE.—In the sentence immediately following Eq. 34, Mr. Knapp states, for a condition of equilibrium: "The depth along the outer wall should be $h_o + \frac{V^2 b}{2 r g}$." The next sentence is considered as meaning that the depth calculated by Eq. 32*b*, for conditions of nonequilibrium, amounts to nearly $h_o + 2\left(\frac{V^2 b}{2 r g}\right)$. Mr. Knapp then concludes: "The disturbance pattern that oscillates about the equilibrium depth, therefore, has a wave length of $2\theta_o$ and an amplitude of $\frac{V^2 b}{2 r g}$."

On the basis of the foregoing statements, the writer infers that the depth of water, at the position of first maximum depth, may correctly be taken as the sum of the normal depth, h_o, and an amount of superelevation equal to $\frac{V^2 b}{r g}$; and that the value so obtained will not be substantially different from that value calculated by the equation derived from wave mechanics. As a matter of fact, the validity of the expression, $\frac{V^2 b}{r g}$, as a measure of maximum superelevation for supercritical flow, is not essentially dependent upon proof by comparison. The value can be obtained by equating expressions for work and potential energy without requiring specific knowledge concerning wave fronts, pressure differences, and other details of the disturbance pattern.

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In Fig. 68, which shows the cross section of a channel entering a circular curve without transition, the line AB represents the water surface of a unit length prism at the end of the tangent. Immediately after the prism passes the beginning of the curve, centrifugal force and wall reactions cause the water surface at the outer wall to rise above point B and the water surface at the inner wall to drop below point A. The center of mass of the prism moves from its original position, E, toward the outer wall. The work being done by the centrifugal force is being stored in the prism as kinetic energy and potential energy. When the cross slope of the water surface reaches the slope for static equilibrium, the kinetic energy is maximum and the movement continues. The maximum depth, $h_o + s$, will be reached when all kinetic energy has been converted to potential energy. At this time, the total work done by the centrifugal force in distorting the unit length prism to its final shape, A′B′CD, will equal the potential energy of the distorted prism with respect to its normal position of rest. Assuming that the mass of the prism can be considered as concentrated at its center, and since, by geometry, the horizontal and vertical distances between the original center, E, and the final center, F, are equal to $\frac{s\,b}{6\,h_o}$ and $\frac{s^2}{6\,h_o}$, respectively, it can be stated that

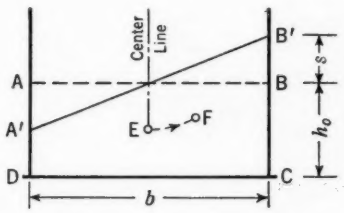

FIG. 68.—CHANNEL SECTION—SUPERELEVATION AT FIRST MAXIMUM DEPTH

$$\frac{m\,V_o^2}{r} \times \frac{s\,b}{6\,h_o} = m\,g \times \frac{s^2}{6\,h_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots (66)$$

$$s = \frac{V_o^2\,b}{r\,g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (67)$$

and

$$h = h_o + \frac{V_o^2\,b}{r\,g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (68)$$

The foregoing derivation assumes that the mean velocity is constant throughout the reach considered and that the radius of the center of mass is that of the channel center line. It also assumes the line A′B′ as straight and as revolving about the intersection of AB and the channel center line. None of these assumptions is exactly correct but all of them are considered to be approximations within the range of permissible error.

One other assumption deserves mention. It is true, as Mr. Knapp states, that the individual filaments are not changed in direction until after reaching the wave front. However, these filaments in the central part of the channel will be deflected later and more sharply than those nearer the walls. Therefore, since the expression for work done is written for the entire reach within the indicated longitudinal limits, it appears reasonable to conclude that the mean value of the summation of all products of masses (m) and the accelerations will be numerically equal to $m\,V_o^2/r$. By analogy, the work performed

in lifting a bag of buckshot through any vertical distance will be equal to the total work done in raising the bag, and then its contents, shot by shot, regardless of variability in the separately applied accelerative and decelerative forces.

Eq. 68 is derived for positions of A′ between A and D, only (see Fig. 68), and should not be used where the flow prism becomes triangular. The writer wonders whether Eq. 32*b* does not become less applicable when the prism is triangular, inasmuch as the author states that "* * * the analysis assumes a true rectangular channel." However, if the general analysis used in the derivation of Eq. 68 be applied to conditions of flow in triangular prisms, it will be found that the maximum depth attains a value of

$$h = \frac{3}{4}\left[(h_o + s) + \sqrt{(h_o + s)^2 - \frac{32}{9} h_o\, s}\,\right] \quad \ldots\ldots\ldots\ldots (69)$$

in which

$$s = \frac{V^2_o\, b}{r\, g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (70)$$

In Mr. Knapp's paper, the position of the first maximum depth is determined by wave pattern geometry. The values obtained from Eq. 33 can be shown to check, quite closely, the values determined from an equation derived by a method involving simple harmonic motion. In Fig. 69(*a*), line AB repre-

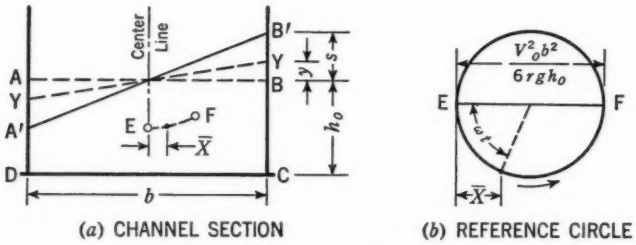

FIG. 69.—POSITION OF FIRST MAXIMUM DEPTH

sents the original water surface; line A′B′, the water surface at the position of first maximum depth; and line Y–Y, the water surface for any intermediate value of superelevation designated as y. At any instant of time, the resultant horizontal force acting upon the prism of unit length equals the centrifugal force minus the net reaction from the walls. For a value of superelevation equal to y, this reaction can be shown to equal $2\, w\, y\, h_o$, in which w is the unit weight of water. The value of the resultant horizontal force can therefore be stated as $w/g\; b\; h_o\; V^2_o/r - 2\, w\, y\, h_o$. This expression when divided by the mass, gives the corresponding acceleration,

$$a = \frac{V^2_o}{r} - \frac{2\, g\, y}{b} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (71)$$

Eq. 71 can also be expressed in terms of the horizontal displacement of the center of mass from its original position, E. This displacement, designated as

$\bar{X}$, equals $\frac{y\,b}{6\,h_o}$. By substitution in Eq. 71,

$$a = \frac{V^2_o}{r} - \frac{12\,g\,h_o\,\bar{X}}{b^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (72)$$

In either Eq. 71 or Eq. 72, the acceleration is zero when the superelevation equals $\frac{V^2_o\,b}{2\,r\,g}$. Either equation obeys the law of simple harmonic motion since the acceleration, on either side of the position of static equilibrium, is directly proportional to the displacement. Therefore, in accordance with the familiar relationship, the horizontal projection of the motion of the center of mass is the projection upon the horizontal diameter of a point moving at constant velocity on the circumference of a circle having a radius of one half the total horizontal displacement. As indicated in Fig. 69(*b*), the radius equals $\frac{V^2_o\,b^2}{12\,r\,g\,h_o}$. At any time, *t*,

$$\bar{X} = \frac{V^2_o\,b^2}{12\,r\,g\,h_o}\,(1 - \cos\omega\,t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (73)$$

and

$$a = \frac{d^2\bar{X}}{dt^2} = \frac{V^2_o\,b^2}{12\,r\,g\,h_o}\,\omega^2 \cos\omega\,t \ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

Equating Eq. 74 to Eq. 71, and imposing the conditions for the total displacement ($t = T$ and $\omega = \pi/T$), the time required for the displacement is

$$T = \frac{\pi\,b}{\sqrt{12\,g\,h_o}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

Therefore, the distance from the beginning of the curve to the position of first maximum depth is equal to

$$L_m = \frac{\pi\,b\,V_o}{\sqrt{12\,g\,h_o}} = 0.16\,\frac{b\,V_o}{\sqrt{h_o}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (76)$$

This derivation has been made under the same general assumptions as described in the text following Eq. 68.

Eqs. 66 to 76 are for circular curves without approach transitions. The oscillations about the equilibrium cross slope are due to the abruptness with which the centripetal forces are applied to the stream filaments. One of the effective methods for the prevention of oscillations is that of using a spiral approach easement, in which the radius decreases at a gradual rate from infinity to that of the circular curve. If this varying radius be introduced in Eq. 72, with the length of spiral expressed as $V\,t$, the resulting differential equation involving $d^2\bar{X}/dt^2$ can be solved to show that the length of spiral required to produce the equilibrium depth at the beginning of the main curve will be twice the value of L_m in Eq. 76. This solution, of course, is subject to such qualifications as may be justified by the approximate nature of the previously described assumptions. Actually, the filaments in the central portion of the flow will not follow the channel curvature until the wave fronts are reached and so absolute equilibrium is not quite achieved.

Comparisons.—The first paragraph of this discussion quoted a statement by Mr. Knapp with regard to the amplitude and the wave length of the oscillation which occurs for supercritical flows on circular curves without approach easements. The writer has endeavored to present a rational proof of Mr. Knapp's evaluation of the amplitude and also, without recourse to wave mechanics, to determine the distance to the position of the first maximum depth, which occurs for a half wave length of the oscillation. Despite his statement relative to the amplitude, Mr. Knapp has presented Eq. 32*b*, and not $h_o + \frac{V^2_o\, b}{r\, g}$, as a measure of total depth. Therefore, it is interesting to compare the numerical results obtainable from the two equations. The solutions of Eq. 33, involved in the solutions of Eq. 32*b*, will also be compared with the values derived from Eq. 76.

TABLE 7.—COMPARISONS OF CALCULATED VALUES

β_o	$\frac{b}{r}$	θ_o	θ'_o	$\frac{s}{s'}$	$\frac{s}{h_o}$	$\frac{s'}{h_o}$	$\frac{s - s'}{h_o}$	$\frac{h}{h'}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
12°	0.02	5° 00′	5° 19′	0.961	0.463	0.481	−0.018	0.988
	0.04	10° 00′	10° 27′	0.900	0.925	1.028	−0.103	0.949
	0.06	....	15° 19′	0.895[a]	1.448[a]	1.618	−0.170[a]	0.935[a]
20°	0.02	3° 23′	3° 07′	1.109	0.171	0.154	+0.017	1.014
	0.04	6° 05′	6° 09′	1.092	0.342	0.313	+0.029	1.022
	0.06	9° 07′	9° 05′	1.080	0.513	0.475	+0.038	1.026
28°	0.02	2° 13′	2° 08′	1.279	0.091	0.071	+0.020	1.018
	0.04	4° 26′	4° 13′	1.280	0.181	0.142	+0.039	1.035
	0.06	6° 38′	6° 15′	1.282	0.272	0.212	+0.060	1.049
36°	0.02	1° 46′	1° 34′	1.532	0.058	0.038	+0.020	1.019
	0.04	3° 32′	3° 05′	1.551	0.116	0.075	+0.041	1.038
	0.06	5° 18′	4° 35′	1.559	0.174	0.111	+0.063	1.056

[a] Computed by the formula for triangular prism flow (Eq. 69), since Eq. 68 cannot be applied when $\sin \beta_o$ is less than $\sqrt{b/r}$.

As shown in Table 7, the comparisons have been made for a wide range of variable conditions. With limits for β_o set at 12° and 36°, provision is made for velocities ranging between 4.8 and 1.7 times the critical velocities. The ratios of b/r cover the cases in which the radii vary between 50 b and 16.7 b. The equations have been compared with respect to θ_o and also for various ratios involving the superelevation, s, and the maximum depth, h. For convenience, the values of θ_o, s, and h as derived from Eq. 33 and Eq. 32*b* are represented as θ'_o, s', and h'.

The values in Table 7 have been obtained from the equations in the following modified forms: Since $L_m = \frac{\pi\, b\, V_o}{\sqrt{12\, g\, h_o}} = \frac{\pi\, r\, \theta_o}{180}$,

$$\theta_o = \frac{51.96\, b/r}{\sin \beta_o} \qquad (77a)$$

and

$$\theta'_o = \tan^{-1} \frac{2\, b/r}{(2 + b/r) \tan \beta_o} \qquad (77b)$$

Since $\sin \beta_o = \sqrt{g\, h_o}/V_o$, Eqs. 67, 68, and 32*b* can be recast into such form as to obtain the following ratios:

$$\frac{s}{s'} = \frac{b/r}{\sin^2 (\beta_o + \theta'_o/2) - \sin^2 \beta_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots (78a)$$

$$\frac{s}{h_o} = \frac{b/r}{\sin^2 \beta_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (78b)$$

$$\frac{s'}{h_o} = \frac{\sin^2 (\beta_o + \theta'_o/2) - \sin^2 \beta_o}{\sin^2 \beta_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots (78c)$$

and

$$\frac{h}{h'} = \frac{b/r + \sin^2 \beta_o}{\sin^2 (\beta_o + \theta'_o/2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (78d)$$

The comparative values for the angular distances to the positions of first maximum depths are shown in Cols. 3 and 4, Table 7. The best agreements are found in the high and middle range velocities. The greatest contrast is for sharpest curvature and slowest velocity where θ_o is 16% larger than θ'_o. The value of θ_o for $\beta_o = 12°$ and $b/r = 0.06$ is omitted since triangular prism flow is beyond the range for the application of Eq. 77*a*.

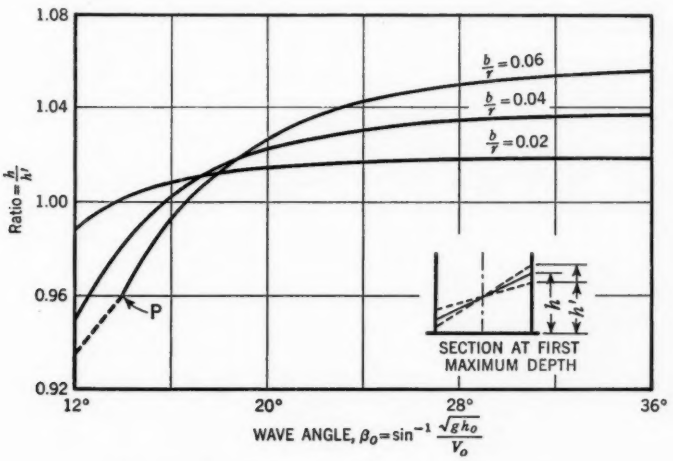

FIG. 70.—RATIO OF CALCULATED MAXIMUM DEPTHS

The ratios of superelevations are entered in Col. 5, Table 7. It will be noticed that the poorest agreements are for the slowest velocities. Although the comparative values for $\beta_o = 36°$ differ quite widely, the differences are not of much practical significance since the superelevations are small. This statement is borne out by the ratios given in Cols. 6, 7, 8, and 9, Table 7. In the entire range studied, values of maximum depth differ by a maximum of about 6%.

The depth ratios shown in Col. 9, Table 7, plus supplementary ratios between $\beta_o = 20°$ and $\beta_o = 12°$, have been plotted in Fig. 70. Equality of

Eq. 32*b* and Eq. 68, for the range studied, lies approximately between wave angles of 14° and 17°. The value of b/r apparently has very little effect upon the ratios when the wave angle equals 18°. The discontinuity at point P, Fig. 70, is due to the substitution of Eq. 69 for Eq. 68.

All the foregoing comparisons have been made with respect to values obtained by calculation. Table 2 furnishes experimental data which permit Eq. 32*b* and Eq. 69 to be compared with reference to observed values of first maximum depth. Table 8 makes this comparison. The values entered in Cols. 1 to 4 and Col. 7 are all obtained from Table 2 or derived from other furnished data. Since the four runs were made for a ratio of $b/r = 0.06$, the values of h/h' in Col. 5, Table 8, are obtained from Fig. 70. Col. 6 shows the values for maximum depth as of Eq. 69. In Cols. 7 and 8, ratios of measured maximum depths to calculated maximum depths are given for Eq. 32*b* and for Eq. 69, respectively.

TABLE 8.—COMPARISONS WITH OBSERVED VALUES

Run No. (Table 2) (1)	β_o (2)	Computed h' (Table 2) (3)	Measured h_m (Fig. 19) (4)	$\frac{h}{h'}$ (Fig. 70) (5)	$h = h' \times \frac{h}{h'}$ (6)	$\frac{h_m}{h'}$ (Table 2) (7)	$\frac{h_m}{h}$ (8)
117C	12° 39′	0.387	0.336	0.943	0.365	0.868	0.921
120A	12° 27′	0.496	0.450	0.940	0.466	0.905	0.968
119D	12° 29′	0.584	0.550	0.940	0.549	0.942	1.002
32A	11° 48′	0.693	0.700	0.932	0.646	1.010	1.084

In closing, the writer wishes to comment upon two of the statements made in the Symposium with respect to banking. Mr. Knapp states (sentence following Eq. 35): "* * * that, with the proper use of banking, all the force necessary to change the direction of the flow is supplied by the cross slope." If "cross slope" means the cross slope of the bottom, this statement is true provided that the slopes of the bottom and the water surface are parallel, and provided that the water prism possesses no transverse momentum due to any lack of equilibrium in upstream sections. On the other hand, if "cross slope" means the cross slope of the water surface, the statement would be true even if the banking be improper or if it be lacking entirely. The centripetal force which changes the direction of flow is the algebraic summation of wall reactions plus the horizontal component of the floor reaction. In the ideal case where the bottom slope is parallel to the water surface the wall reactions will be equal and opposite in direction, and the floor will be furnishing the entire turning force. In the general case (that is, where the banking is not in strict conformity with the ideal requirement), the walls must assist in the turning.

On the practical side, in some instances, the use of banking may have rather serious disadvantages. One disadvantage is that the low water flow is concentrated against the base of the inner wall. In case the flow carries sand and persists for considerable periods of time, erosion of concrete surfaces is likely

to occur. Another disadvantage becomes apparent in those cases in which the channel grade is not steep enough to prevent the occurrence of an adverse grade along the inner wall in the downstream transition.

FRED W. BLAISDELL,[42] ASSOC. M. ASCE.—The papers of the Symposium are a fine addition to the small amount of literature available on high-velocity flow in open channels. They should serve to acquaint those who have never dealt with high-velocity flows with the entirely different methods that must be used handling flows at supercritical velocities as compared with the methods used when the velocity is in the more common and more familiar subcritical range. The presentation, in such a practical way, of information that can be used for the preliminary design and, in some cases, for the final design of hydraulic structures subjected to high-velocity flows is particularly welcome to the designer who has had to guess rather than to estimate what to expect both qualitatively and quantitatively and who has undoubtedly guessed wrong at times because of the lack of information such as is presented herein.

Comments on Theory.—The writer is favorably impressed with Mr. Ippen's efforts to make the descriptions of the mechanics of supercritical flow both clear and complete. Fig. 3, relating the deflection angle with the Froude number and the relative depth, should be particularly useful in preliminary investigations. It should be remembered that, even if Eqs. 10*a* and 10*b* can be used to give the depth, they must be used with a plan of the structure, and also with Eq. 5 to determine where these depths occur. The graphical method of characteristics, of course, gives both the depth and its location simultaneously and is to be preferred when determining the flow surface if it is safe to assume that there is no energy dissipation.

Mr. Ippen's statements (under the heading, "Mechanics of Wave Propagation in Supercritical Flow: Wave Interference") to the effect that a successful analysis of certain problems requires a fair prediction of the initial Froude number $\mathbf{F}_1$ is interesting. The writer interprets this to mean that a fairly good determination of $\mathbf{F}_1$ must be made. Theoretically it seems that the exact value of $\mathbf{F}_1$ would be required. However, since the simplifying assumptions used in developing the theoretical solution are at variance with what actually occurs, extreme efforts to secure an exact determination of $\mathbf{F}_1$ could hardly be justified. Nevertheless, some effort should be made to secure the best possible value of $\mathbf{F}_1$ for even the best may not be too good; the most probable value of $\mathbf{F}_1$ should be used, not the maximum or minimum expected values. The selection of the proper friction factor is only one obstacle to an exact determination of $\mathbf{F}_1$, and model tests are of little help here because the scaling up of many depths depends on the selection of the proper friction factor in both the model and its prototype. It should be realized that an error in determining $\mathbf{F}_1$ results in a shift in location of the disturbance lines and may cause the counterdisturbance source, which would theoretically cancel the original disturbance, to be improperly located. Although the counterdisturbance, unless grossly mislocated, should have some beneficial effect in reducing the size of the original disturbance, efforts should be

[42] Project Supervisor, Soil Conservation Service, U. S. Dept. of Agriculture, St. Anthony Falls Hydr. Laboratory, Minneapolis, Minn.

made to locate the counterdisturbance as nearly as possible in its correct position and this requires the determination of the most likely value of $\mathbf{F}_1$.

The graphical method of characteristics is certainly a valuable aid to the solution of problems involving disturbances in high-velocity flows. It is possible, as Mr. Ippen states, to solve problems quite easily once the fundamental procedures are understood. In fact, in less than a week the writer trained a subprofessional draftsman to solve problems of channel expansions by the method of characteristics. As a result of this experience the writer prepared detailed instructions for the use of the method of characteristics in solving problems involving expanding transitions.[43] However, these instructions were based on the paper by Ernst Preiswerk,[9] published in 1940. Although identical results can be obtained with both Mr. Ippen's method[10] and Mr. Preiswerk's method, the latter uses the critical depth $2\,H/3$ as his reference head instead of the specific head H used by Mr. Ippen. This results in Mr. Preiswerk's epicycloids being drawn between the limits $\sqrt{3}\,\bar{V} = 1$ and $3\,\bar{V} = \sqrt{3}$.

In Fig. 7 Mr. Ippen shows a scale of h/H which may be computed from Eq. 13. A scale of $\bar{V}$ can also be added to Fig. 7 although the writer has found little need for it. However, the writer has found a scale of $\mathbf{F}$, which can also be added to Fig. 7, to be most useful. In fact, the writer's instructions call for scales of h/H, $\bar{V}$, and $\mathbf{F}$ engraved on a transparent sheet. The equation for $\bar{V}$ in terms of $\mathbf{F}$ is derived as follows: By definition, $\bar{V} = V/\sqrt{2\,g\,H}$ and $\mathbf{F} = V/\sqrt{g\,h}$. Solving for V, and equating,

$$V = \bar{V}\sqrt{2\,g\,H} = \mathbf{F}\sqrt{g\,h} \qquad (79)$$

Solving for $\bar{V}$, and substituting for h/H its value as given in Eq. 13, it is found that

$$\bar{V} = \sqrt{\frac{\mathbf{F}^2}{2 + \mathbf{F}^2}} \qquad (80)$$

In practice the writer prefers to construct a family of epicycloids for each problem so that the ellipse will lie over, or equidistant from, the intersections of the epicycloids rather than near the intersections as Mr. Ippen states in the paragraph following Eq. 14. This greatly eases the graphical solution and increases its accuracy. The starting point for the family of epicycloids is found by measuring the distance $\bar{V}$ (or h/H or $\mathbf{F}$) from the origin of the diagram parallel to the initial flow direction. The first epicycloids are drawn in both directions through this point, the remaining epicycloids being drawn at equal angles apart. A typical solution is shown in Fig. 71. It seems likely that Mr. Ippen himself uses this method since in Fig. 38 of the third paper the characteristic diagram apparently was especially constructed instead of being a part of the diagram shown in Fig. 7.

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[43] "Graphical Construction of the Flow in Transitions by the Method of Characteristics," by Fred W. Blaisdell, *Manuscript Report MN-R-3-24*, SCS, U.S.D.A., in cooperation with the Minnesota Agri. Experiment Station and the St. Anthony Falls Hydr. Laboratory, Univ. of Minnesota, Minneapolis, Minn., May, 1944 (revised February, 1946).

[9] "Application of the Methods of Gas Dynamics to Water Flows with Free Surface," by Ernst Preiswerk, *Technical Memoranda Nos. 934 and 935*, National Advisory Committee for Aeronautics, Washington, D. C., March, 1940.

[10] "Gas-Wave Analogies in Open-Channel Flow," by A. T. Ippen, *Proceedings*, 2d Hydraulics Conference, *Bulletin No. 27*, Univ. of Iowa Studies in Eng., Iowa City, Iowa, 1942, p. 257, Fig. 3.

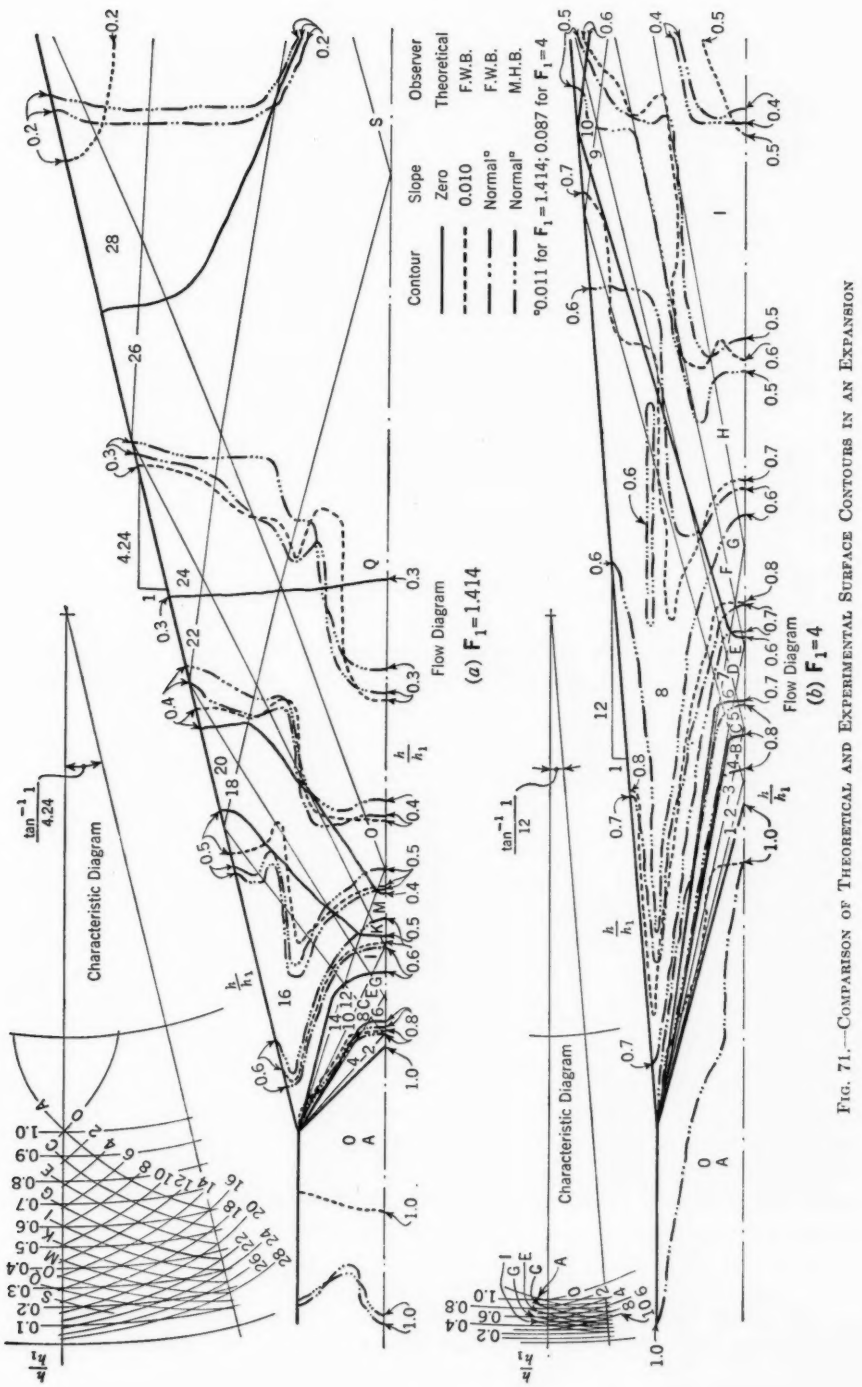


FIG. 71.—COMPARISON OF THEORETICAL AND EXPERIMENTAL SURFACE CONTOURS IN AN EXPANSION

It is interesting to note that each point on the coordinate system of Fig. 38(*a*) has a corresponding point on the coordinate system of Fig. 38(*b*). This permits an instant determination of the value of h/H at any point in Fig. 38(*a*) simply by referring to the correspondingly numbered point in Fig. 38(*b*). For example, it can be seen that in area F_o, $h/H = 0.234$ which has been plotted at Stations 1, 3, 5.5, etc., in Fig. 38(*c*). From Fig. 38(*b*) it can be seen that $h/H = 0.234$ also in areas C_6, D_4, E_2, etc. Incidentally, the theoretical surfaces in Figs. 38(*c*) and 38(*d*) could have been drawn as smooth lines instead of stepped lines; the steps are a result of the method of solution since θ was varied in finite steps—that is, the increments were in terms of $\Delta\theta$ rather than $d\theta$. However, the vertical lines representing the shock-wave fronts would necessarily be retained.

It would be helpful if Mr. Ippen would define the magnitude of the "small error" which, as he states, results when Eq. 19*b* is substituted for the exact form (Eq. 19*a*).

Channel Junctions.—It is too much to expect that this Symposium, composed of four papers treating only as many different phases of the problem, could be an "inclusive treatise" on the subject of high-velocity flow in open channels, as was hoped in the "Foreword." One major problem which the authors have not mentioned is the joining of two streams, one or both of which may be flowing at supercritical velocities. Several problems of this type were studied on models during late 1948 and 1949 by C. E. Bowers, Jun. ASCE, formerly research associate at the St. Anthony Falls Hydraulic Laboratory of the University of Minnesota, in Minneapolis, under the general supervision of the writer.

At Whiting Field Naval Auxiliary Air Station, near Pensacola, Fla., it was proposed to construct paved trapezoidal open channels to conduct the drainage water from the plateau on which the airfield is located. The flow in many of these channels is at supercritical velocities. At various points most of the channels pick up additional water from terraces and lateral channels. The problem of joining the two streams was subjected to model study and several successful solutions were obtained Four of these solutions will be described.

Both Mr. Ippen (under the subheading, "Wave Interference") and Mr. Knapp (in the "Synopsis" and under the subheading, "Interference Treatments") have suggested that one disturbance may be used to cancel another disturbance. This method was tried at one junction during the exploratory stage of the study; a channel flowing at a supercritical velocity was joined by a channel flowing at a subcritical velocity, a diverter pier was inserted in the main channel, and the diverted flow was reflected from the vertical wall as shown in Fig. 72. That this reflected wave largely canceled the wave caused by the flow entering from the lateral can be seen in Fig. 73(*a*). The waves in the downstream channel before the insertion of the pier are shown in Fig. 73(*b*) for purposes of comparison. Greatly improved flow conditions in the downstream channel were obtained for all flows as long as the flows from the two channels were aproximately proportional to their design flows.

The design recommended for this particular junction, which originated with Mr. Bowers, is shown in Fig. 74(*a*) and the resulting flow conditions are shown

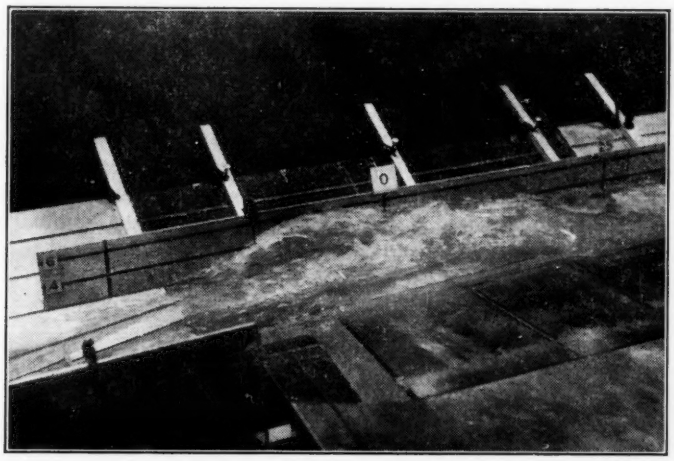
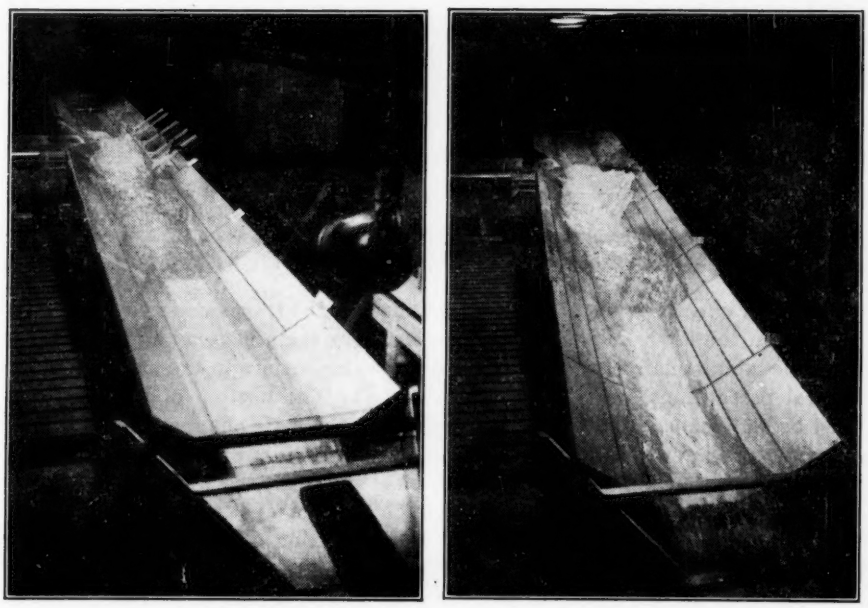


FIG. 72.—COUNTERDISTURBANCE PIER TYPE OF CHANNEL JUNCTION

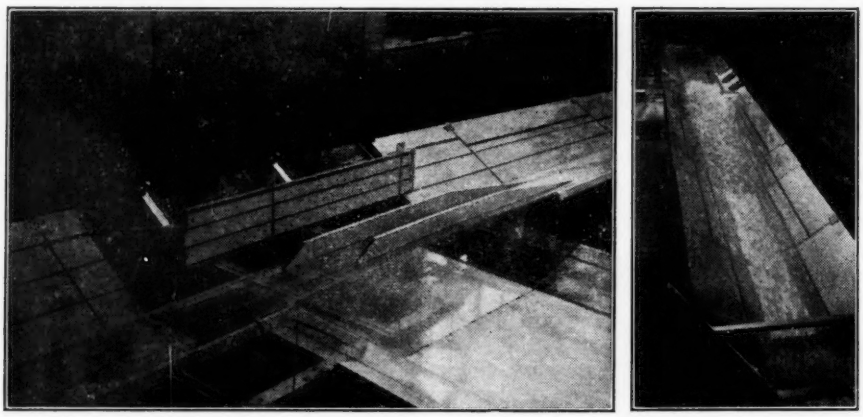


(*a*) Counterdisturbance Pier Design Almost Completely Cancels Out Cross Waves

(*b*) Serious Waves Are Present When a Simple Junction is Used

FIG. 73.—FLOW CONDITIONS IN CHANNEL DOWNSTREAM FROM JUNCTION; CONTOURS ARE AT 2-FT VERTICAL INTERVALS

in Fig. 74(*b*). The long low piers which are submerged at the design flow have much the same effect as do the multiple curved vanes mentioned by Mr. Knapp. They are remarkably effective in eliminating disturbances in the downstream channel at the design flow as well as at flows less than the design value, and for hydrographs which are so out of synchronism that there may be little or no flow from one channel while the other is flowing at its design capacity. For any particular combination of flows from the channels the piers could be much shorter than those shown in Fig. 74(*a*). However, for other combinations of flows the pier locations would have to be shifted along the channel to be of maximum effectiveness. The result is that long piers are required to cover the anticipated range of flow conditions.



(*a*) Piers and Vertical Wall Opposite Junction

(*b*) Flow Conditions in Channel Below Junction

FIG. 74.—SUBMERGED PIER TYPE OF JUNCTION DESIGN; CONTOURS ARE AT 2-FT VERTICAL INTERVALS; HEAVY LINE REPRESENTS TOP OF RECOMMENDED SIDE WALL

Another effective type of junction is known as the transverse weir type. In this design the lateral channel is carried over the main channel and the water from the lateral channel is dumped onto the water surface in the main channel. Several precautions are required, however: (1) The bottom of the "bridge" must be high enough above the water surface in the main channel so that waves and surges will never strike the transverse weir structure and cause a hydraulic jump to form that might overtop the channel sides; (2) the width of the nappe where it strikes the water surface in the main channel must be fully equal to the width of the water surface to prevent the formation of strong disturbances; (3) the ratio of the flow over the weir to the flow in the main channel should be quite small, probably; and (4) the Froude number in the main channel must be high enough to insure that the velocity will not decrease from supercritical to subcritical at the junction. Regarding precaution (3), ratios tested for different locations at Whiting Field had values between 0.16 and 0.06. From what little information is now available, it appears that this ratio should be less than 0.08. Regarding precaution (4), Froude numbers between 2.9 and 4.0 gave satis-

factory results. A test with a Froude number of 1.7 and a discharge ratio of 0.06 yielded results that were barely acceptable. At two transverse weir junctions it was necessary to recommend a local steepening of the channel grade in order to increase the Froude number so that satisfactory flow conditions could be obtained. In addition to these precautions, the use of one of more of the submerged piers just described may be desirable to improve flow conditions further. The transverse weir type of junction originated with Arthur F. Moratz, head, District Operations Design and Construction, Soil Conservation Service, Milwaukee, Wis., who was in charge of the design of the structures used in the drainage system at Whiting Field.

In cases where the Froude numbers of the joining streams are low it may not be possible to join them at supercritical velocities. One junction was developed for use at Whiting Field where the Froude numbers were 2.8 and 3.9 in the entering main and lateral channels, respectively. For this junction the flows were passed through the hydraulic jump upstream from the junction, the streams joined at subcritical velocities, and the flow was accelerated to supercritical velocities downstream from the junction.

Channel Expansions.—In the "Introduction," the authors of the fourth Symposium paper state that the graphical method is limited in its application to expanding channels because it offers no clue as to the stability or instability of a hydraulic jump at the downstream end of the expansion. The writer feels that, if the graphical method of characteristics can be used to predict the surface levels in the transition with reasonable accuracy, then it is not limited in its application. This is all that the method of characteristics is supposed to do. The stability or instability of the hydraulic jump as defined in the fourth Symposium paper may result from the nonuniform depths at the end of the transition; but the performance of the jump is a separate, although related, problem that is beyond the scope of the method of characteristics.

In 1944 the writer and his associates compared selected cross sections of a jet expanding freely on a level floor with the cross sections determined by the method of characteristics. The results of this study throw some light on the effect that the assumptions used in developing the method of characteristics have on its practical application. The experimental data used in this study were obtained by Mr. Bhoota[30] at the State University of Iowa, in Iowa City. Presumably a part of this material is presented in Fig 54. The complete data cover streams having initial cross section width-depth ratios of 2, 4, and 8 and initial Froude numbers increasing in geometric progression by $\sqrt{3}$ from 1 to 27. Good agreement was obtained between the theoretical and experimental cross sections. The average difference at 78 cross sections between the data obtained by Mr. Bhoota and that computed by the method of characteristics was only 2% of the initial cross-sectional area. Although one difference was as high as 27%, there were only three differences greater than 9%, and all of these were for the highest Froude number where the greatest difficulty could be anticipated in obtaining accurate water-surface measurements due to the rough surfaces that

[30] "Characteristics of Supercritical Flow at an Abrupt Open-Channel Enlargement," by B. V. Bhoota, thesis presented to the State University of Iowa at Iowa City, Iowa, in December, 1942, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

undoubtedly existed. In general the areas obtained by Mr. Bhoota increased relative to the characteristic areas with distance along the jet. This tendency could possibly be ascribed to the effect of friction, which is neglected in the theoretical analysis. Nevertheless, it appears that the comparison of the theoretical and experimental results for this particular set of data is excellent.

The writer has been interested in expanding transitions since discovering some years ago that the use of expansions ahead of a stilling basin can produce significant economies in the cost of the complete structure.[44] The studies on transitions which followed this discovery covered the determination of the best side-wall flare for expansions having straight side walls and also the determination of surface contours and flow properties along transitions laid on a slope of 1% where the side-wall flare[44] was 1 in 3 **F**. The reason for selecting this side-wall flare is that the tests showed the most rapid side-wall flare to be 1 in 3 **F** to insure a reasonably good surface configuration and eliminate the higher waves. Although arrived at independently, this is also the flare recommended by Mr. Bhoota.[30] Subsequently, the writer conducted a few tests with the channel floor laid on the normal slope for the approaching stream and these tests were continued by Merlin H. Berg, Assoc. M. ASCE, in 1948 as a thesis study.[45]

To compare the experimental data with the theoretical predictions the writer has computed, graphically, the theoretical surface profiles for two values of **F**. The surface contours resulting from these computations are presented in Fig. 71. Also plotted in Fig. 71 are surface contours obtained by the writer on 1% and normal slopes and those obtained by Mr. Berg using normal slopes. The work of two observers is included to show approximately what precision can be expected when two supposedly identical test runs are made. Some of the contours have been omitted to avoid confusion. In studying Fig. 71 it should be remembered that the water-surface slopes are quite flat relative to the channel floor, especially for the lower relative depth contours, and that a considerable horizontal displacement in the position of any two contours denoting a given depth may indicate only a small difference between the depths at any specific point.

Surface contours for those transitions located on normal slopes would presumably agree better with the theoretical predictions than do those contours obtained when the floor is level. This is because (as noted by Mr. Ippen, under the subheading, "Basic Properties of Standing Waves," and by Mr. Knapp when discussing assumptions (*c*) and (*d*), under the heading, "Calculation of Liquid Surface Profiles Along the Walls") the normal slope should largely compensate for the friction and produce, in effect, flow on the frictionless surface used in developing the theory. It can be seen in Fig. 71(*b*) that the agreement between the theoretical and experimental contours is considerably improved if the transition is located on a normal slope. The difference between the normal and 1% slopes in Fig. 71(*a*) is not sufficient to cause a significant difference between the contours there. The writer is unable to understand why the authors of the fourth Symposium paper did not obtain similar results.

[44] "Flow Through Diverging Open Channel Transitions at Supercritical Velocities," by Fred W. Blaisdell, *Technical Publication No. 76*, SCS, U.S.D.A., Washington, D. C., April, 1949.

[45] "Supercritical Flow in Straight-Wall Diverging Channels on Normal Slopes," by Merlin H. Berg, thesis presented to the University of Minnesota, at Minneapolis, Minn., in August, 1948, in partial fulfilment of the requirements for the degree of Master of Science.

Eq. 48 is presented as the most satisfactory form of the boundary curve; further discussion of the reasons for selecting this particular curve would be helpful. A question that arises in the mind of the writer is: Do the disturbance lines originating on one side of the transition strike the opposite wall, or is the wall curve formulated so as to eliminate the possibility of reflections?

As several of the authors have noted, much remains to be learned concerning flow at supercritical velocities. However, this should in no way detract from the fine beginning made by the Symposium authors.

J. H. DOUMA,[46] Assoc. M. ASCE.—Heretofore, hydraulic design of channel expansions, contractions, and bends has been based all too often on unproved empirical or rule-of-thumb criteria, not subjected to critical theoretical analysis. In some cases this results from a lack of research and development information; in other cases there seems to be lacking, also, an appreciation of the need for fundamental information on the part of personnel responsible for design.

In the third Symposium paper Messrs. Ippen and Dawson state that straight-wall contractions are the most satisfactory for supercritical flow in open channels. Past practice has conformed with this criterion in the case of contractions which are required when right-of-way restrictions or changes in invert grade of a concrete-lined, high-velocity channel make it economical to reduce the channel width. Here, invert grades are relatively mild, varying from about 0.5% to 3%, and contractions vary from about 10% to 25% of the channel widths. The practice has been to design straight-wall contractions of generous length to provide small angles of divergence, thereby causing the formation of small transverse waves that are confined within normal freeboard allowances. There is need for a simplified criterion to determine minimum desirable lengths of contractions for any combination of channel dimensions and flow elements, which it may be possible to develop from the data presented.

Another common type of channel contraction is encountered in the design of chute spillways, wherein it is usually economical to contract from a wide upstream section to a narrow downstream section. In this case, the width if control section or gate structure is determined by discharge requirements and economic considerations, and the width of downstream chute, by topography, character of foundation, and economical channel dimensions. The length of contraction, which is designed to provide efficient passage of flow from the control structure to the downstream chute, is dependent upon the percentage of contraction (which is sometimes as much as 300%, as in the case of transition from a gate structure 120 ft wide to a chute 40 ft wide) and upon the permissible height of transverse waves in the chute spillway.

When the channel downstream from a contraction is short, or when chute spillways discharge directly into side canyons, it is less costly to design the contraction with a large over-all angle of convergence and to provide greater wall heights to offer protection against overtopping by the resulting higher waves; but, when the downstream channel is long and overtopping by waves is likely to endanger the structure, it will be economical to provide a long contraction so that waves will be contained within the normal freeboard allowance. Infre-

[46] Hydr. Engr., Office, Chf. of Engrs., Washington, D. C.

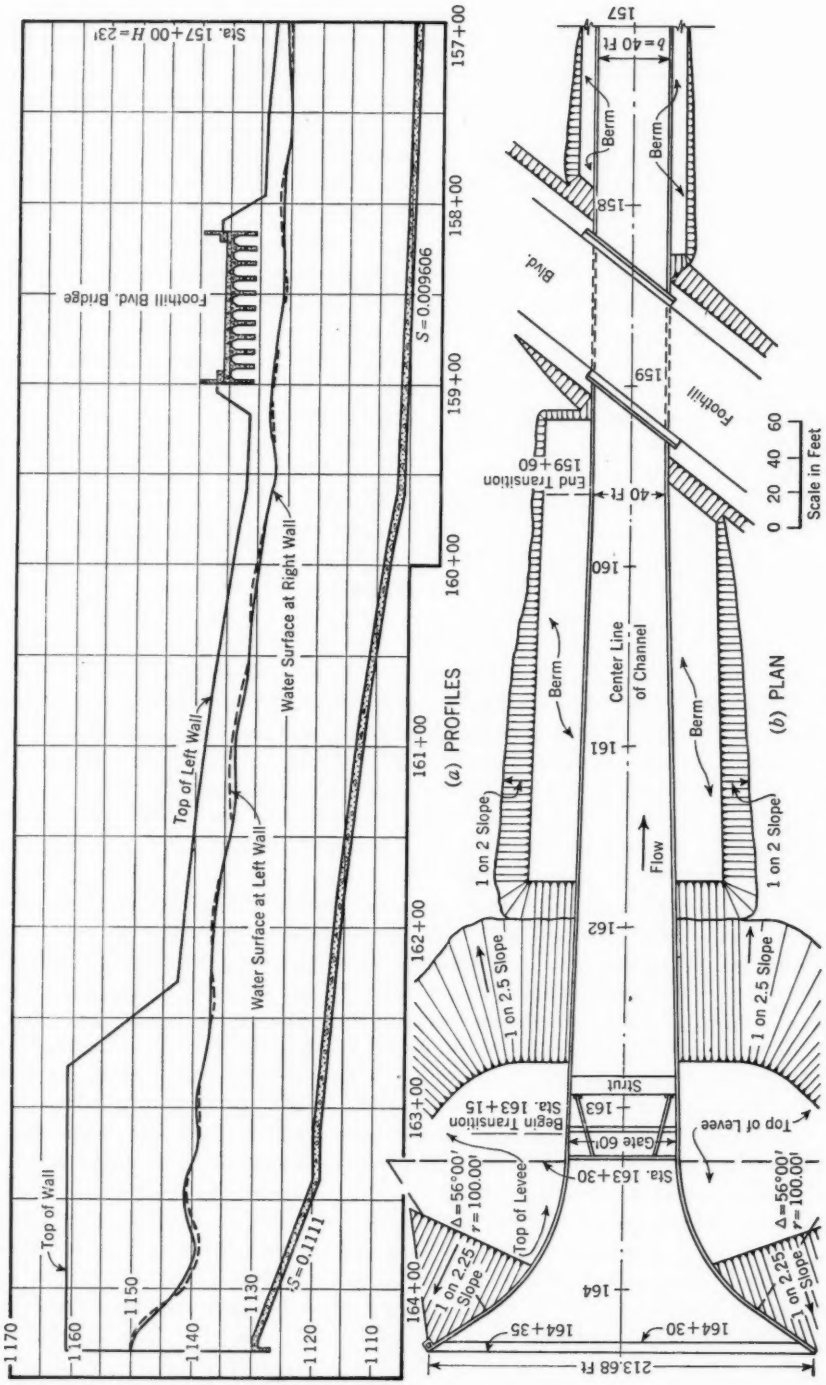


FIG. 75.—INTAKE OF FLOOD CONTROL CHANNEL, LYTLE CREEK, CALIFORNIA

quent overtopping by waves may be permitted when the downstream channel is located in deep cut or when the downstream channel is founded on firm rock.

A third type of channel contraction, a combined intake and drop structure located at the upstream end of a flood control channel, is a long, efficient contraction which confines waves to the improved channel at minimum cost. This type is illustrated by the Lytle Creek (California) flood-control channel for which hydraulic model tests indicated that the intake contraction would function best if the side walls were curved upstream from the control to provide a large percentage of the total contraction in low-velocity sections of the intake and if the side walls were made straight downstream from the control to provide a small convergence angle for high-velocity flow. The plan and profile of the Lytle Creek intake are shown in Fig. 75. This design almost completely eliminated transverse waves in the contraction and uniform channel sections downstream from the control. Uniform flow depths and symmetrical velocity distributions throughout the contraction are illustrated in Figs. 76 and 77, respectively.

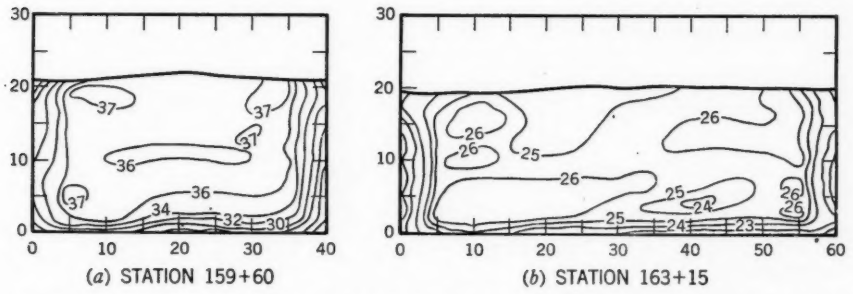


FIG. 76.—VELOCITY DISTRIBUTION AS MEASURED IN A 1:16 SCALE MODEL OF THE LYTLE CREEK (CALIFORNIA) CONTRACTION. (DISTANCES ARE IN FEET AND VELOCITIES ARE IN FEET PER SECOND)

Lateral expansions, which are discussed in the fourth Symposium paper, are encountered most frequently in concrete-lined, high-velocity channels with mild slopes, and in outlet channels with steep slopes, located downstream from reservoir conduits. Generally, expansions on mile slopes are designed sufficiently long to insure that waves will be confined within the normal freeboard allowance. This practice does not materially increase costs, inasmuch as the total length of channel improvement is not increased; but, in the case of outlet channels, long expansions do increase costs.

Although the general relationship presented by Eq. 48 should provide a basis for determining the optimum length of expansion when the invert slope is mild, it is questionable whether the relationship will apply in the case of a steep slope, as it does not contain a slope factor. It is believed that the rate of the spreading of a high-velocity jet decreases as the invert slope increases. In the extreme case, if no invert were provided and the jet issued into free air, the rate of spread would be small compared to the corresponding rate on a mild slope. Eq. 48 would be more widely applicable if it were modified to meet this requirement.

Mr. Knapp states, in the second Symposium paper, that compound and spiral transition curves prevent the formation of wave disturbances in channel bends and downstream tangents when flow is supercritical. However, as the experiments conducted at the California Institute of Technology did not include velocities lower than those corresponding to a Froude number of 1.5, it is not known whether the formulas presented for correction by compound curves are applicable when the flow is near that of the critical depth. Interference wave patterns produced by compound curves would be greatly complicated by undulating waves characteristic of flow at or near critical stage. Moreover, superelevation of the water surface is not eliminated, and attendant nonuniform depths of flow, caused by centrifugal force when the invert is not banked, require variable wall heights throughout the length of curve and unequal heights of the two walls at common stations, unless the inside wall is made higher than required by the design water surface to maintain equal heights.

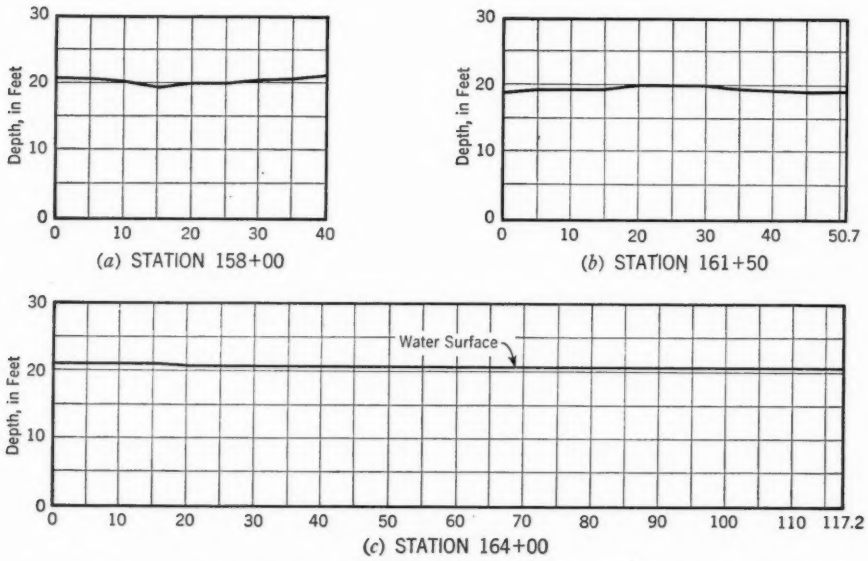


FIG. 77.—TRANSVERSE WATER-SURFACE PROFILES AS MEASURED IN A 1:16 SCALE MODEL OF THE LYTLE CREEK (CALIFORNIA) CONTRACTION

Nonuniformity of wall heights may be eliminated by banking the invert so that it is constantly parallel to the transverse slope of the design water surface in accordance with Eq. 35.

In order to produce gradual changes in flow, a transition, consisting of spiral curved walls and an invert whose transverse slope varies linearly from zero at the beginning of the spiral to the transverse slope required for equilibrium along the circular curve at the end of the spiral, is required upstream and downstream from each curve. On the basis of theoretical analysis by the Los Angeles County Flood Control District, the minimum length of spiral, L_s, required in precluding the formation of transverse waves in a rectangular curved

reach may be expressed by

$$L_s = 1.82 \frac{b\,V}{(g\,d)^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (81)$$

in which d is the design depth of flow.

Although, theoretically, banking the invert for the design discharge will produce the desired correction without the use of spiral transition curves, for lower and higher discharges the transverse invert slope is incorrect, which results in flow disturbances. Furthermore, on the basis of large-scale model

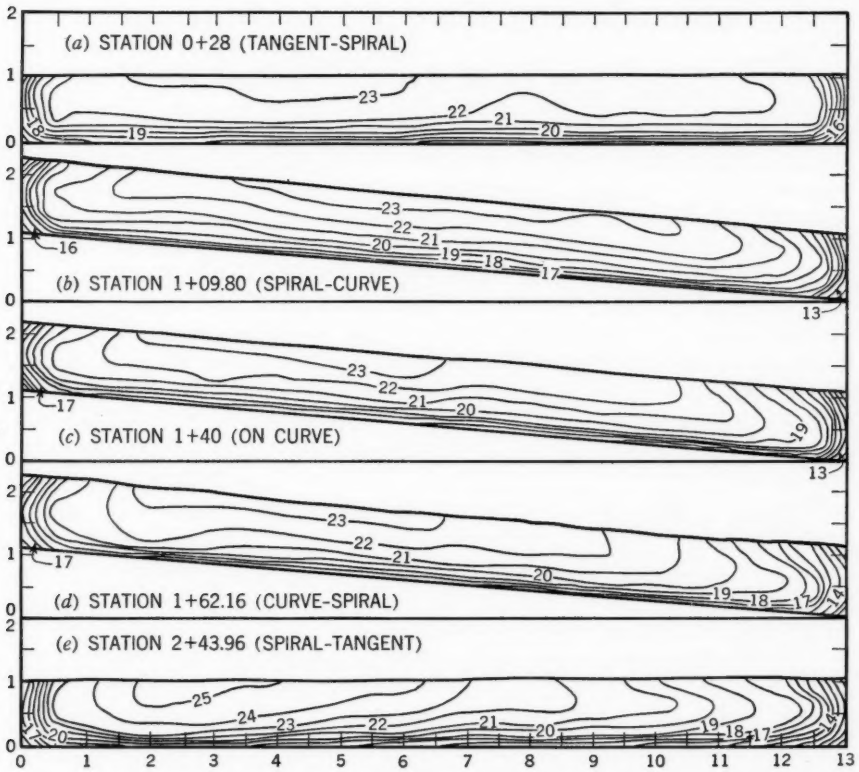

(a) STATION 0+28 (TANGENT-SPIRAL)
(b) STATION 1+09.80 (SPIRAL-CURVE)
(c) STATION 1+40 (ON CURVE)
(d) STATION 1+62.16 (CURVE-SPIRAL)
(e) STATION 2+43.96 (SPIRAL-TANGENT)
0 1 2 3 4 5 6 7 8 9 10 11 12 13

FIG. 78.—VELOCITY DISTRIBUTION FOR SUPERELEVATED FLOW IN A CURVED CHANNEL (RADIUS, 150 FT; WIDTH, 13.0 FT; VELOCITY, 20.5 FT PER SEC; DISCHARGE, 280 CU FT PER SEC; DISTANCES ARE IN FEET; AND VELOCITIES ARE IN FEET PER SECOND)

tests conducted by the Los Angeles District, Corps of Engineers, unless spiral transitions are used in a curved reach of short radius, wave disturbance occurs for the design discharge. For projects in which the channel alinement contains a number of curves in series and flow is near that of the critical depth (as is the case of many channels in the Los Angeles area), it is considered important to eliminate wave disturbances for all flows, whenever practicable, because of the potential synchronizing effect of wave disturbances in successive curves.

The results of this series of large-scale model tests (which were conducted to determine the adequacy of design criteria for superelevated curves of rectangular channels in which the flow velocity is greater than critical) indicate that the design criteria are satisfactory for all channels in which the ratio of velocity head to the center-line radius of the curve does not exceed 0.04. Examination of Fig. 78 shows that: (1) The depth of flow is nearly uniform across the channel throughout the curved reach; (2) in all sections, the water surface is approximately parallel to the banked invert; and (3) transverse surface waves do not form in the curved reach. As a result of the model tests, the design criteria were considered adequate and curved reaches of rectangular concrete flood control channels are now being designed with spiral transitions and banked inverts.

On the basis of detailed, comparative estimates of cost made by the Los Angeles District, Corps of Engineers, for wide (from 60 ft to 130 ft) rectangular concrete channels with high walls (from 15 ft to 20 ft), relatively small radii of curvature (from 500 ft to 1,500 ft), and high velocities (from 30 ft per sec to 40 ft per sec), banked inverts and spiral transition curves result in a saving of 10% in construction costs. The estimates show that the additional cost of constructing banked inverts and spiral transitions, rather than horizontal inverts and simple or compound curves, are small compared to the saving effected by lower walls of uniform height.

Application of the analyses and procedures suggested, in the Symposium, to the design of high-velocity channels, will result in improved design criteria, although (as in the case of any rapidly expanding pioneer field) some of the basic assumptions and formulas may have to be revised later to conform with new developments.

ARTHUR T. IPPEN,[47] M. ASCE.—The discussions submitted in response to the Symposium on "High-Velocity Flow in Open Channels" show a gratifying amount of additional confirmation of the theories and ideas given in that Symposium. Particularly valuable are the comments from practical engineers concerning past and present design procedures.

Mr. Douma mentions the practice of designing straight-wall contractions of generous length to provide small angles of convergence. Extra length is not necessarily the answer to small disturbances since, in accordance with Fig. 9(*a*), the saving is minimized with additional length due to reflections within the transition which in turn will cause continuously increasing depth, although the wave height may remain small. This increase of depth, of course, is often prevented by providing additional bottom gradient and thus continuously accelerated flow.

Simplified criteria for dimensions of channel contractions are not easily derived since, in contrast to gradual channel expansions, the wave angle for large disturbances depends on wave height and Froude number and since energy dissipation may have to be considered. However, rapid approximations can be made if, in accordance with notations in Fig. 41(*b*), the front ABC′D′ is

[47] Prof. of Hydraulics, Dept. of Civ. and San. Eng., Massachusetts Inst. of Technology, Cambridge, Mass.

assumed straight and if the angle of β is initially approximated from sin $\beta_1 = 1/F_1$. Then $\tan \beta_1 = \dfrac{b_1 - (b_1 - b_2)}{L}$ or

$$L = \frac{b_1 + b_2}{2 \tan \beta_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (82)$$

With this value of L, an approximate value of θ is obtained from

$$\tan \theta = \frac{b_1 - b_2}{2\,L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (83)$$

Starting from this initial value of θ, the correct length L can then be evolved by means of Fig. 8 or the corresponding equations.

It is to be noted that a maximum angle of deflection is associated with every Froude number and that two deflections take place in each contraction. If flow conditions below fronts AB and A′B in Fig. 41(*b*) cannot sustain a second deflection through angle θ, a normal hydraulic jump must form at the intersection of the wave fronts in B which will then move upstream beyond line AA′. Supercritical flow through the contraction is then impossible. In other words, in accordance with Fig. 8, the value of θ required would exceed the maximum permissible deflection under the fronts BCD and BC′D′.

Ample confirmation of the theory expressed in summary by Fig. 8 is now available from extensive experimental work at the Hydrodynamics Laboratory of the M.I.T. (Cambridge, Mass.) for a large range of Froude numbers and deflection angles.[48]

The application of any formula presented to conditions near critical depth is open to serious question since, by nature of this flow, any approximations made in the theory are no longer permissible and since the smallest disturbance will cause extremely large undulations. For this reason, any transferability of model results to prototypes must also be in doubt.

Eq. 81 concerning the length of the spiral approach is not the result of any new theory but is an approximation for the half-wave length required by the theory given and modified on the basis of experiments. Banking must always be accompanied by transition sections for the confining walls. Whether these confining walls in a transition are of the spiral, circular, or any other type, the essential criterion for the counterdisturbance is: (1) That the length of the transition be one half the wave length; and (2) that the total deflection at the end of the transition be one half of the central angle of a corresponding length of curve with normal radius.

It is only necessary to produce a counterdisturbance of the proper magnitude which, in accordance with Fig. 3, does not depend on the curvature of the channel, but only on the total deflection produced within the transition.

There is complete agreement with Mr. Baumann and Mr. Douma that banking should be the normal solution for important projects. There is no reason, however, why compound curves as proposed in the Symposium should

[48] "Studies on the Validity of the Hydraulic Analogy to Supersonic Flow," by Arthur T. Ippen and Donald R. F. Harleman, *Technical Report No. 5985*, U. S. Air Force, October, 1949, Pts I and II.

not have extensive application whenever banking is not feasible on account of structural or economic conditions.

That banking does not eliminate all disturbances is well brought out by Mr. Douma in Fig. 78 where, for some reason, the velocity distribution upon entering the curve, Station 0 + 28, is uneven. Considerable deceleration is indicated at Station 1 + 09.80 when the maximum velocity shifts away from the outside. This tendency would have been more pronounced if the velocity distribution at Station 0 + 28 had been symmetrical about the axis. Considerable acceleration takes place between Station 1 + 62.16 and Station 2 + 43.96 along the outside, resulting in considerable differences in velocity for the downstream tangent of the curve between outside and inside walls. This effect of uneven kinetic energy distribution may cause serious difficulties with respect to producing a uniform hydraulic jump if a stilling pool follows downstream.

The theories dealing with transverse oscillations of flow in curved channels presented by both Mr. Baumann and Mr. Hart must be recognized as applicable only to large-radius curves. They impose a serious limitation by their assumption of a uniform transverse surface gradient which is not found in practice. The treatment is strictly applicable only to supercritical flow in curves and therefore does not contribute to the basic understanding of supercritical flow in general, which should be acquired by all those responsible for design work in this field.

The introduction of curvature $1/r$ into any of the problems is not very helpful, and Eq. 68 fails, for example, if $1/r$ approaches infinity. Eq. 10*a*, which is identical with Eq. 28 and which is graphically represented in Fig. 3, has no such shortcoming. Thus the importance of recognizing the primary role of wall deflection angles and the nature of propagation of transverse disturbances is emphasized again. Also the expressions given hold for curved wall sections of any length where as Eq. 68 applies only for lengths greater than one half of the wave length. The concept of centrifugal force in connection with any of the supercritical flow problems is not a fruitful one since fluid particles are not rigidly tied to each other and, therefore, methods dealing with the bulk flow are incorrect.

Mr. Blench refers to the relatively poor agreement between theory and experiment in the example given in Fig. 38. The surface curves agree excellently within the contraction itself, and there exists no other method of determining such surface profiles than the one given. In the downstream tangent, the agreement becomes rapidly worse due to the limiting factors discussed before—primarily, the effect of successive intersection of steep waves and the cumulative side-wall shear. The theory given for sudden changes in surface elevation summarized in Fig. 8 takes full account of energy losses.

Mr. Bhandari suggests a comparison between free surface transitions for supercritical flow and those in closed conduits. The physical behavior of the two types of flow is entirely different. In closed conduits and in subcritical flow contracting channel walls without change in bottom gradient cause accelerated flow and expansions produce deceleration. For supercritical flow, the reverse is true, contracting flow causes deceleration, and expanding flows

cause acceleration as clearly shown in papers 3 and 4. There exists no geometric similarity for flows in the same transitions for different Froude numbers due to widely varying standing wave patterns.

Mr. Blaisdell's discussion contains excellent confirmation of the theory presented and suggests only minor variations in technical procedure. Special thanks are due him for his clear presentation of additional supercritical flow variants. The writer does not agree with respect to the increased accuracy claimed in applying the method of characteristics. It seems that a densely drawn network of epicycloids should make interpolation easy when the values of $\overline{V}$ do not correspond to those given along an epicycloid. To draw special diagrams for each particular problem seems unnecessary and would not result in increased accuracy beyond that warranted by the basic assumptions of the theory.

The example given concerning the use of submerged piers to produce interference waves is very interesting. Naturally, any design of a supercritical flow structure can be correct only for a certain value of the Froude number. Model results for higher Froude numbers are readily transferable to prototype conditions if Froude number values are the same. Since in most practical cases these values are somewhat in doubt, model tests must be expanded to include possible variations. The Reynolds numbers of effects in almost all studies have been found to be negligible except when long channels with successive wave intersections and reflections are encountered.

A very effective contribution to the Symposium has been made by discussing various types of channel junctions, and the methods presented for eliminating disturbances in these cases are ingenious as well as helpful for future designs.

Toward the development of more rapidly expanding flows than are possible by side-wall flare and uniform bottom gradient, the "humping" of the bottom or the use of transverse slopes from the center toward the sides may be mentioned. Such a problem was solved successfully by the writer through model tests at Lehigh University (Bethlehem, Pa.) on the Clark Creek Spillway which was constructed in 1940, and a uniform jump below the expansion was obtained.

In conclusion, the writer would like to express his appreciation for the notable contributions made by the discussers and hopes that a helpful beginning has been made in developing rational design procedures which will be applied by fellow engineers engaged in the design of hydraulic structures for supercritical flow.

ROBERT T. KNAPP,[49] M. ASCE.—Several assumptions are made in connection with the derivation referred to in the paper and the deviations from actual conditions involved in some of them are quite serious. These deviations have been re-emphasized by Mr. Baumann. It was the writer's intention to state the nature of the assumptions clearly so that the user might be in a position to evaluate, for each application, whether or not the assumptions are valid within the required accuracy. It is believed that in most cases the other uncertainties in the design data are so large that refinements in the assumptions made in the paper will not be warranted.

[49] Director, Hydrodynamics Laboratory, California Inst. of Technology, Pasadena, Calif.

Mr. Baumann presents an ingenious method for deriving the maximum height and the position of the first maximum in the curve. However, the validity of the physical analogy upon which the method is based is not completely clear. The agreement of most of the laboratory results with Eq. 32*b* and the agreement of most of the field results with Eq. 61*b* is probably more fortuitous than real because of the deviation in both cases of the real conditions from the assumptions of the flow characteristics.

Mr. Baumann's discussion of the cost of building the spiral transitions is very informative. It appears that there is little to choose between spiral and compound transitions from the point of view of cost. Therefore, since both accomplish the same end result hydrodynamically, it would appear that the choice between them is a matter of individual taste. Mr. Baumann's comments regarding banked channels is very interesting. As stated in the paper, they are an elegant solution, particularly for channels operating mainly near design capacity. Mr. Baumann also comments upon the inadvisability of using sills in channels carrying debris. The writer would agree that they are less desirable in debris-carrying water, but it should be remembered that the recommended installation leaves free spaces between both ends of the sill and the channel walls. Since there is a vigorous circulation along the upstream face of each sill, it is doubtful if debris can collect there during a major flow. It is believed that the deposits which will undoubtedly be found above each sill at the end of a storm period are formed during the receding stages of the flood when the need for the sill action is no longer present. However, it should be remembered that sills are recommenedd primarily as remedial measures, not for the design of a new channel.

It is not surprising that Mr. Hart finds other methods of deriving some of the equations than those employed by the authors of the Symposium. There are usually many physically correct approaches to the analysis of a physical problem, and all correct approaches will result in the same answer within the limits of the simplifying assumptions made in each. The writer is disturbed, however, at Mr. Hart's reluctance to accept and utilize the concepts of wave motion to simplify, not only the calculations, but also the physical concepts of the mechanics of the flow. The foundations of wave mechanics are identical with those of other phases of mechanics. Wave motion is a classical example of simple harmonic motion.

Mr. Hart and a few others seem to share an erroneous impression concerning the validity of the use of wave mechanics to treat the problem of high-velocity flow in open channels. They imply that the justification for the use of the methods of wave mechanics is only that the high-velocity flow bears a resemblance to some wave problems. This falls far short of the truth. The fact is that high-velocity flow in open channels is a wave problem and that the equations derived for supersonic flow in gases are also the equations that express the physical realities of the open-channel flow. Hydraulic engineers should be thankful that so much work has been done in theoretical wave mechanics in other fields of science and engineering so that the prior findings can be applied with confidence to hydraulic problems without retracing all the laborious steps that have brought them to their present useful state.

One further point raised by Mr. Hart is the writer's use of the word "cross slope" with reference to banking of the channel bottom. He is correct in assuming that the word refers to the cross slope of the bottom. In his discussion of the action of the cross slope of the water surface as compared to the cross slope of the bottom, however, it appears that there is some confusion between cause and effect.

Mr. Douma calls attention to the difficulty of working with flows whose Froude numbers are in the vicinity of 1. This is a "cross" that must be borne. It corresponds directly to the dreaded "sonic barrier" which plagues the aerodynamicists. Gross approximations are usually not adequate for work in this region so that much more knowledge is needed in this area of open-channel flow before such problems can be handled with confidence. One of the chief difficulties is that the flow energies for the subcritical and supercritical regimes in this region differ so little that the flow is inherently unstable and may swing from one regime to the other with the slightest disturbance.

In the discussion of the use of banking of the channel bottom, Mr. Douma states:

> "Although, theoretically, banking the invert for the design discharge will produce the desired correction without the use of spiral transition curves, * * *."

The writer cannot agree with this statement if it implies that a correctly designed wall transition curve is not required as a part of the design of a banked curve. As was pointed out in the discussion of banking, adjacent to Eq. 35, the radius of curvature of the walls should decrease as the bottom cross slope increases and should always conform to the curvature of the flow produced by the bottom slope at each point on the transition. If this procedure is not followed, the walls will do some of the turning of the flow with the development of the usual pattern of cross waves. Of course, it would be possible to design deliberately to do part of the turning of the flow by the bottom cross slope and part by the walls of the transition, but this appears to be a needless complication in normal cases.

The writer wishes to thank all the discussers for their interest and helpful suggestions as well as for their kind comments regarding the value of the Symposium.

HUNTER ROUSE,[50] M. ASCE, AND B. V. BHOOTA[51] AND EN-YUN HSU,[52] ASSOC. MEMBERS, ASCE.—As is often true of comments on such relatively new concepts as were presented in this Symposium, the discussions ranged from a keen appreciation of the principles involved to a total disregard for the pertinent differences between supercritical and subcritical flow. It is therefore particularly heartening that the majority of the discussers, by actual use of the concepts, had already been convinced of their fundamental importance in hydraulic design.

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Mr. Baumann, who was among the first in the United States to realize the necessity of designing for high-velocity flow by new methods rather than standard methods, comments on the simplicity of the writers' correlation of expansion characteristics according to the Froude number. Mr. Bhandari, on the other hand, presumes—the theory of supercritical flow to the contrary—that the same angle of permissible divergence which applies to closed-conduit expansions and to open-channel expansions for subcritical flow should also be applicable to open-channel expansions for supercritical flow. The writers have shown both analytically and experimentally—and their findings are substantiated by Mr. Blaisdell's discussion—that such is by no means the case; in fact, this distinction between the two flow regimes is the very crux of the Symposium. (In view of these circumstances, it is almost impertinent to note in addition that even in the case of closed conduits or of open channels for subcritical flow there is no fixed angle below which separation will not occur, since the limiting angle, despite Mr. Nikuradse's earlier indications, has been shown by more recent boundary-layer analysis to decrease with increasing length of transition.)

Mr. Blench makes a noteworthy point in regard to separation tendencies when he points out that supercritical flow in an expansion is inherently accelerative whereas subcritical flow in an expansion is decelerative—and it is in zones of deceleration that boundary-layer separation is to be expected. However, what is called separation in supercritical flow is not primarily a boundary-layer effect (although the latter may be involved to some degree), but rather the result of wall deflection at a greater rate than the finite celerity of wave propagation will permit the flow at once to follow. Such local phenomena of wave propagation are definitely three-dimensional, although each paper of the Symposium simplifies them to two dimensions for ease in analysis. The work of Mr. Hinds, to which Mr. Blench refers, further simplifies the treatment to a single dimension. Whereas the latter simplification is very useful for subcritical conditions, it can yield significant results for supercritical conditions only if the flow actually possesses the comparatively rare one-dimensional characteristics—that is, continuous convergence or divergence at a constant angle without beginning or end effects. One-dimensional analysis of this nature can obviously give no clue as to the local effects of wave formation, interference, and reflection which formed the basis of the papers under consideration. On the other hand, the two-dimensional wave analysis permits just as ready inclusion of shock losses if they are actually involved. In reply to Mr. Blench's question as to the possibility of designing an expansion with constant depth across each normal section, this must be answered categorically in the negative, if only because of the local variations essential to wave propagation. It is possible, to be sure, to design an expansion which will produce a constant depth across the outlet, as discussed in the paper. At intermediate sections, however, the desired constancy can only be approached as a limit by making the expansion uneconomically long and gradual. As far as conditions in the vicinity of the critical (chosen by Mr. Blench for comparison purposes) are concerned, these are difficult to fit very satisfactorily into as simple a generalization as the writers proposed, if only because of the appreciable curvilinear

(that is, nonhydrostatic, or three-dimensional) effects which come into play as this limit is approached.

Mr Blaisdell's verification, not only of the writers' experimental results but also of the agreement between the experiments and the simplified theory, is almost too good to expect; in any event it should be most encouraging to those faced with the necessity of using the methods proposed. The writers therefore again call attention to the fact that incorporation of such factors as bed resistance and slope in the analysis represents at best a very rough approximation, since such flow is highly nonuniform and any gross correction of this nature can be based only on average conditions. The situation becomes more inexact as the surface contours for various widely different flow conditions are superposed as in the writers' further simplification, for the effects of bed resistance and slope then change not only from point to point and section to section but also from case to case. Rather similar circumstances may be cited in answer to Mr. Blaisdell's query as to the form of the recommended wall curve. For a given case, the continuous curve which the disturbance lines from the opposite wall just touch (or just miss) probably represents an optimum theoretical solution for the given Froude number. Such a curve is not algebraically expressible, however, and superposition of results in the writers' manner for various Froude numbers would yield as many different curves. The single power function which the writers used in their investigations is merely a convenient approximation which was selected, for the various reasons cited, upon a purely experimental basis.

Mr. Douma is quite correct in presuming that relationships determined for a very small slope will be of little use if the slope is great. As he surmises, and as indicated by the writers in their brief discussion of this factor, the optimum rate of expansion for a given Froude number will necessarily decrease more and more rapidly as the slope increases. Since the method of including slope as a primary variable in the wave theory is still undeveloped, the only solution which the writers can suggest is the preparation, based on experimental measurement, of a series of interpolation diagrams similar to Fig. 57 for successively greater slopes. Although it is questionable whether even approximate superposition of curves could then be obtained, the procedure would at least yield preliminary design information and should also provide a quantitative basis for eventual further analysis.

The writers join with the authors of the other Symposium papers in expressing to the discussers their appreciation for the interest which they have shown and the additional contributions which they have made. Since new engineering concepts are invariably slow to be accepted, capable discussions of this nature often prove as useful to the profession as the papers themselves.

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